ON RAYLEIGH – TAYLOR INSTABILITY IN MAGNETOHYDRODYNAMICS IN THE GALVANIC APPROXIMATION

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The paper considers the stability of the interface between viscous conducting media in the presence of a current and a magnetic field for a magnetic Reynolds number much smaller than unity. In order to obtain the dispersion relationships, a method is used that is based on the variational principle. It is shown that the method indicated yields good results when the magnetic field is considered. The dependence of the maximum growth rate of the instability on the defining parameters is presented. The problem of the stability of a fluid layer situated between solid walls for linearly distributed conductivity and density is likewise solved. The stabilizing effect of the Hartmann number on the stability is shown.

A number of papers have been devoted to an investigation of stability of the Rayleigh-Taylor type in magnetohydrodynamics for a finite conductivity of the medium (for example, [1-4]). In the majority of these papers the stability of the interface between media having different densities and conductivities was considered in the presence of gravitational and magnetic fields, the external magnetic field being assumed constant (i.e., in the equilibrium state the currents were assumed equal to zero).

The assumption of a constant magnetic field, which permits analytic solutions of the problem to be obtained in a number of cases, excludes the interesting case of the stability of a current slab. In the general case the problem of oscillations in a thin slab having a finite conductivity of the medium turns out to be very complex [4].

At the same time, in a number of cases which are of practical interest the effect of the magnetic field introduced by the current flowing in the fluid is relatively small, and the problem of the stability of a current slab in gravitational and magnetic fields may be solved in the galvanic approximation. Such an approach was initially proposed in [5], where dispersion relationships were derived for bulk and surface waves in the absence of viscosity or an external electric field.

In [6] the problem of the stability of the interface between two media having different conductivities was considered with allowance for the viscosity of the medium and the finiteness of the conductivity gradient. However, that paper did not consider the current induced by the perturbed motion of the medium, although there were not sufficient grounds for this omission; also, at the same time the perturbations of the electric field intensity were assumed to be nonvanishing, which is not correct in the indicated statement of the problem. In [7] an attempt was made at obtaining the galvanic approximation via the transition to the limit $R_m \to 0$ in the final equations for the perturbations: under these conditions the electric field perturbations automatically dropped out, but the magnetic field perturbations were preserved.

Finally in [8] only dispersion relationships derived for $R_m \ll 1$ for two particular cases were given: in the absence of viscosity of the medium and for a small interaction parameter. Moreover, in that paper the magnetic field perturbations were considered anew. The indicated factors prompt another consideration of this problem.

1. Let us consider the stability of an infinite plane slab of incompressible, viscous, and conducting fluid situated in contact either with analogous slabs characterized by other values of density, viscosity, and

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conductivity or with walls. A gravitational field having a constant acceleration $g(0, 0, -g)$ is applied normally to the unperturbed boundary. A uniform magnetic field B is directed along the x axis, and an electric field E_0 is directed along the y axis. Here and henceforth the subscript 0 denotes the parameters corresponding to the unperturbed equilibrium state.

The original system of equations has the following form in the generally accepted notation:

$$\begin{gathered}\rho\frac{\partial \mathbf{V}}{\partial t}+\rho(\mathbf{V}\nabla)\mathbf{V}=-\nabla p+\nabla(\mu\nabla\mathbf{V})+(\nabla\mu)(\nabla\mathbf{V})+\rho\mathbf{g}+\mathbf{j}\times\mathbf{B}\\ \operatorname{div}\mathbf{V}=0,\quad \mathbf{j}=\sigma(\mathbf{E}+\mathbf{V}\times\mathbf{B}),\quad \operatorname{rot}\mathbf{E}=0,\quad \operatorname{div}\mathbf{j}=0\end{gathered}\tag{1.1}$$

If diffusion is neglected, then one may write the equation

$$\frac{\partial\rho}{\partial t}+(\mathbf{V}\nabla)\rho=0$$

for the density.

Assuming the conductivity and viscosity to depend only on density, we obtain the following equations:

$$\frac{\partial\sigma}{\partial t}+(\mathbf{V}\nabla)\sigma=0,\quad \frac{\partial\mu}{\partial t}+(\mathbf{V}\nabla)\mu=0$$

The equilibrium state for which $\mathbf{V}_0 = 0$ and the parameters ρ_0, σ_0 and μ_0 are functions of only one coordinate z is determined by the equation

$$\frac{dp_0}{dz}=-\rho_0 g-\sigma_0 E_0 B$$

In order to investigate the stability of such a state, we introduce small perturbations of all the parameters in the form $f(z)\exp(ik_x + ik_y + nt)$.

After linearizing the original system (1.1), we may reduce it to one equation for the perturbation of the velocity component along the z axis:

$$\begin{gathered}n\left[\rho_0 W-\frac{1}{k^2}D(\rho_0 DW)\right]+\frac{k_x^2}{k^2}\sigma_0 B^2 W+\frac{1}{k^2}D\left[\mu_0(D^2-k^2)DW\right]\\ +\frac{1}{k^2}D\left[D\mu_0(D^2+k^2)W\right]-\mu_0(D^2-k^2)W-2D\mu_0 DW\\ -\frac{1}{n}\left(gD\rho_0+\frac{k_x^2}{k^2}E_0 BD\sigma_0\right)W=0\end{gathered}\tag{1.2}$$

Here $k^2 = k_x^2 + k_y^2$, $D \equiv d/dz$.

This equation must be supplemented by the boundary conditions which have the same form as they do in the absence of the magnetic field [10], since the presence of the latter did not increase the order of the equation. Thus, we have either $W = DW = 0$ on a solid boundary or at infinity (if an unbounded medium is considered), or $W = D^2W = 0$ on a free surface; we likewise have continuity of W, DW, $\mu_0(D^2 + k^2)W$ and

$$\left[\rho_0-\frac{\mu_0}{n}(D^2-k^2)\right]DW+\frac{2k^2}{n}\mu_0 DW+\frac{1}{n^2}(gk^2\rho_0+k_x^2E_0B\sigma_0)W$$

on the interface between slabs.

2. Let us go on to consider the problem of the stability of the interface between two semiinfinite nonviscous media with constant densities and conductivities in gravitational and magnetic fields. From Eq. (1.2) we easily obtain the following result with allowance for the boundary conditions at infinity

$$W^{(1)}=Ce^{\alpha_1 z},\quad W^{(2)}=Ce^{-\alpha_2 z}$$

where

$$\alpha_i^2=\frac{k_x^2B^2\sigma_0^{(i)}}{n\rho_0^{(i)}}+k^2,\quad \operatorname{Re}(\alpha_i)>0,\quad i=1,2$$

Using the conditions for the continuity of the velocity and pressure on the interface between media, we obtain the following conversion relationship:

$$n^2\left[\rho_0^{(1)}\left(\frac{k_x^2B^2\sigma_0^{(1)}}{n\rho_0^{(1)}}+k^2\right)^{1/2}+\rho_0^{(2)}\left(\frac{k_x^2B^2\sigma_0^{(2)}}{n\rho_0^{(2)}}+k^2\right)^{1/2}\right]$$
$$=k^2g\left(\rho_0^{(2)}-\rho_0^{(1)}\right)+k_x^2E_0B\left(\sigma_0^{(2)}-\sigma_0^{(1)}\right) \tag{2.1}$$

In the case when the conductivity of one of the media is equal to zero ($\sigma_0^{(1)} = 0$) Eq. (2.1) may be reduced to an equation of fourth degree in n:

$$n^4-\frac{k_x^2}{k^2}\,\frac{B^2\sigma_0^{(2)}\rho_0^{(2)}}{\rho_0^{(1)2}-\rho_0^{(2)2}}\,n^3-\frac{2\left[g\left(\rho_0^{(2)}-\rho_0^{(1)}\right)k^2+k_x^2E_0B\sigma_0^{(2)}\right]\rho_0^{(1)}}{k\left(\rho_0^{(1)2}-\rho_0^{(2)2}\right)}\,n^2$$
$$+\frac{k^2}{\rho_0^{(1)2}-\rho_0^{(2)2}}\left[g\left(\rho_0^{(2)}-\rho_0^{(1)}\right)+\frac{k_x^2}{k^2}E_0B\sigma_0^{(2)}\right]^2=0 \tag{2.2}$$

For $\rho_0^{(1)} = 0$ with $E_0 = 0$ this equation can be reduced to the equations derived in [5] in the absence of a magnetic field component along the z axis.

Thus, even a comparatively simple problem leads to the necessity of its numerical solution. Therefore, approximate methods deserve attention which allow a fairly simple solution of the problem stated in cases which are more general than the one just considered in the sense of boundary conditions and the initial distribution of the unperturbed quantities.

3. As far back as 1883, in [9] the variational principle was used as the basis for proposing an efficient method of solving problems of the stability of an ideal fluid. In [10] this method was generalized for tha case of viscous fluids. Below, the method indicated is generalized for the case of conducting liquids for $R_m \ll 1$.

Let us return to Eq. (1.2). Assume that the eigenvalue n_i corresponds to the eigenfunction W_i, while the eigenvalue n_j corresponds to the eigenfunction W_j. Then, multiplying Eq. (1.2), which has been written for the case i by W_j and integrating within the limits of the flow region considered, we shall have the following equation after integrating the individual terms by parts and considering the boundary conditions:

$$-n_i\int\rho_0\left[W_iW_j+\frac{1}{k^2}(DW_i)(DW_j)\right]dz+\frac{g}{n_i}\int(D\rho_0)W_iW_jdz$$
$$+\frac{1}{n_i}\,\frac{k_x^2}{k^2}E_0B\int(D\sigma_0)W_iW_jdz=\frac{k_x^2}{k^2}B^2\int\sigma_0W_iW_jdz \tag{3.1}$$
$$+\int\mu_0\left[k^2W_iW_j+2(DW_i)(DW_j)+\frac{1}{k^2}(D^2W_i)(D^2W_j)\right]dz+\int(D^2\mu_0)W_iW_jdz$$

Here and henceforth the integration method, which coincides with the boundaries of the flow region, is dropped.

Now, replacing the subscript i by j and substracting Eq. (3.1) from the equation obtained in this fashion, we shall have

$$(n_j-n_i)\left\{\int\rho_0\left[W_iW_j+\frac{1}{k^2}(DW_i)(DW_j)\right]dz+\frac{g}{n_in_j}\int(D\rho_0)W_iW_jdz+\frac{E_0B}{n_in_j}\,\frac{k_x^2}{k^2}\int(D\sigma_0)W_iW_jdz\right\}=0 \tag{3.2}$$

Writing (3.1) in the form

$$g\int(D\rho_0)W_iW_jdz+\frac{k_x^2}{k^2}E_0\int(D\sigma_0)W_iW_jdz$$
$$=n_i^2\int\rho_0\left[W_iW_j+\frac{1}{k^2}(DW_i)(DW_j)\right]dz+n_i\frac{k_x^2}{k^2}B^2\int\sigma_0W_iW_jdz$$
$$+n_i\int\mu_0\left[k^2W_iW_j+2(DW_i)(DW_j)+\frac{1}{k^2}(D^2W_i)(D^2W_j)\right]dz$$
$$+n_i\int(D^2\mu_0)W_iW_jdz$$

and performing analogous operations, we find

$$(n_i + n_j)\int \rho_0 \Big[W_i W_j + \frac{1}{k^2}(DW_i)(DW_j)\Big] dz + \frac{k_x^2}{k^2} B^2 \int \sigma_0 W_i W_j dz$$
$$+ \int \mu_0 \Big[k^2 W_i W_j + 2(DW_i)(DW_j) + \frac{1}{k^2}(D^2 W_i)(D^2 W_j)\Big] dz \tag{3.3}$$
$$+ \int (D^2\mu_0) W_i W_j dz = 0$$

Let us now assume that n_i and n_j are complex-conjugate quantities; then they will correspond to complex-conjugate values of W_i and W_j. Under these conditions it will follow from (3.2) that

$$\mathrm{Im}\,(n) \Big\{\int \rho_0 \Big[|W|^2 + \frac{1}{k^2}|DW|^2\Big] dz + \frac{1}{|n|^2}\int \Big[g(D\rho_0) + \frac{k_x^2}{k^2} E_0 B (D\sigma_0)\Big] |W|^2 dz\Big\} = 0$$

i.e., if

$$g(D\rho_0) + \frac{k_x^2}{k^2} E_0 B (D\sigma_0) > 0$$

then n cannot be complex.

In analogous fashion we have

$$2\,\mathrm{Re}\,(n)\int \rho_0 \Big[|W|^2 + \frac{1}{k^2}|DW|^2\Big] dz = -B^2 \frac{k_x^2}{k^2}\int \sigma_0 |W|^2 dz$$
$$-\int \mu_0 \Big[k^2 |W|^2 + 2|DW|^2 + \frac{1}{k^2}|D^2 W|^2\Big] dz - \int (D^2\mu_0)|W|^2 dz$$

from (3.3), whence it follows that if $D^2\mu_0 \geq 0$, then $\mathrm{Re}(n) < 0$; while if $D^2\mu_0 < 0$ and $|D^2\mu_0|$ is fairly large, then the case $\mathrm{Re}\,(n) > 0$ is possible.

Assuming $i = j$ in (3.1), we obtain

$$n \int \rho_0 \Big[W^2 + \frac{1}{k^2}(DW)^2\Big] dz - \frac{g}{n}\int (D\rho_0) W^2 dz - \frac{E_0 B}{n}\,\frac{k_x^2}{k^2}\int (D\sigma_0) W^2 dz =$$
$$= -B^2 \frac{k_x^2}{k^2}\int \sigma_0 W^2 dz - \int \Big\{\mu_0 \Big[k^2 W^2 + 2(DW)^2 + \frac{1}{k^2}(D^2 W)^2\Big] + (D^2\mu_0) W^2\Big\} dz \tag{3.4}$$

Equation (3.4) allows a variational principle analogous to the one considered in [10] to be used to determine the eigenvalues n. Let us find the deviation δn of the eignevalue n caused by a small deviation δW, which satisfies the boundary conditions. With an accuracy of up to quantities of second-order smallness we shall have

$$-\Big(I_1 + \frac{g}{n^2} I_2 + \frac{1}{n^2} I_3\Big)\delta n = n\delta I_1 - \frac{g}{n}\delta I_2 - \frac{1}{n}\delta I_3 + \delta I_4 + \delta I_5 \tag{3.5}$$

where

$$I_1 = \int \rho_0 \Big[W^2 + \frac{1}{k^2}(DW)^2\Big] dz, \quad I_2 = \int (D\rho_0) W^2 dz$$
$$I_3 = \frac{k_x^2}{k^2} E_0 B \int (D\sigma_0) W^2 dx, \quad I_4 = \frac{k_x^2}{k^2} B^2 \int \sigma_0 W^2 dz$$
$$I_5 = \int \Big\{\mu_0 \Big[k^2 W^2 + 2(DW)^2 + \frac{1}{k^2}(D^2 W)^2\Big] + (D^2\mu_0) W^2\Big\} dz$$

δI_i are the corresponding deviations of the integrals

$$\frac{1}{2}\delta I_1 = \int \Big[\rho_0 W - \frac{1}{k^2} D(\rho_0 DW)\Big]\delta W dz, \quad \frac{1}{2}\delta I_2 = \int (D\rho_0) W \delta W dz$$
$$\frac{1}{2}\delta I_3 = \frac{k_x^2}{k^2} E_0 B \int (D\sigma_0) W \delta W dz, \quad \frac{1}{2}\delta I_4 = \frac{k_x^2}{k^2} B^2 \int \sigma_0 W \delta W dz \tag{3.6}$$
$$\frac{1}{2}\delta I_5 = \frac{1}{k^2}\int \{\mu_0 (D^2 - k^2) W + 2(D\mu_0)(D^3 - k^2) DW$$
$$+ (D^2\mu_0)(D^2 + k^2) W\}\delta W dz$$

Substituting Eqs. (3.6) into Eq. (3.5) and carrying out simple transformations, we obtain

$$
\begin{aligned}
-\frac{1}{2}\Big(I_1+\frac{g}{n^2}I_2+\frac{1}{n^2}I_3\Big)\delta n = \int\Big\{& n\rho_0 W-\frac{g}{n}(D\rho_0)W \\
&-\frac{n}{k^2}D(\rho_0 DW)-\frac{E_0 B}{n}\frac{k_x^2}{k^2}(D\sigma_0)W+\frac{k_x^2}{k^2}B^2\sigma_0 W \\
&+\frac{1}{k^2}D[\mu_0(D^2-k^2)DW]+\frac{1}{k^2}D[D\mu_0(D^2+k^2)W] \\
&-\mu_0(D^2-k^2)W-2(D\mu_0)(DW)\Big\}\delta W dz
\end{aligned}
\tag{3.7}
$$

Now using the original equation (1.2) we shall have

$$
\Big(I_1+\frac{g}{n^2}I_2+\frac{1}{n^2}I_3\Big)\delta n=0
$$

whence it follows that $\delta n = 0$, since the expression in the parentheses is not equal to zero in the general case.

Consequently, small deviations of the eigenfunction δW from its true value, which satisfy the boundary conditions, correspond to the value of n [determined from Eq. (3.4)] with an accuracy up to quantities of second order smallness.

4. Let us go on to consider the problem of the stability of the interface ($z = 0$) between viscous conducting fluids bounded by solid walls ($-h^{(1)}$, $h^{(2)}$), using the variational principle expounded above. As is shown in [11], the application of the variational principle for the solution of the problem stated in the absence of a magnetic field (when the viscosities of the fluids are different) is inexpedient. However, in the statement considered one may limit the analysis to the case when the viscosities of the fluids are identical and the fluids differ only in the magnitudes of their conductivities; this may hold, for example, in a nonuniformly heated gas whose conductivity depends very strongly on temperature, while the density and viscosity vary substantially less. Note that such a statement of the problem makes no sense in the absence of a magnetic field. As an approximate expression for the velocity perturbation we shall take its well-known exact value in the case of an ideal fluid in the absence of a magnetic field:

$$
W^{(1)}=A\left(e^{-kz}-e^{2kh^{(1)}+kz}\right),\qquad W^{(2)}=A\,\frac{1-e^{2kh^{(1)}}}{1-e^{-2kh^{(2)}}}\left(e^{-kz}-e^{-2kh^{(2)}+kz}\right)
$$

Substituting these expressions into Eq. (3.4) and integrating, we obtain the following dispersion relationships:

$$
\begin{aligned}
&\rho_0(\operatorname{cth}kh^{(1)}+\operatorname{cth}kh^{(2)})n^2+\Big\{\frac{k_x^2}{k}B^2\Big[\frac{1}{2k}(\sigma_0^{(1)}\operatorname{cth}kh^{(1)}+\sigma_0^{(2)}\operatorname{cth}kh^{(2)}) \\
&-\frac{2e^{2kh^{(1)}}}{(e^{2kh^{(1)}}-1)^2}(h^{(1)}\sigma_0^{(1)}+h^{(2)}\sigma_0^{(2)})\Big]+2k^2\mu_0(\operatorname{cth}kh^{(1)}+\operatorname{cth}kh^{(2)})\Big\}n \\
&-\frac{k_x^2}{k^2}E_0 B(\sigma_0^{(2)}-\sigma_0^{(1)})=0
\end{aligned}
\tag{4.1}
$$

Assume now that $h^{(1)}, h^{(2)} \to \infty$ and the fluids are nonviscous; then (4.1) takes the form

$$
N_1^2+{}^1/_2 N_1-K_1=0
$$

where

$$
N_1=\frac{2\rho_0 n}{(\sigma_0^{(1)}+\sigma_0^{(2)})B^2},\qquad K_1=\frac{2\rho_0 E_0(\sigma_0^{(2)}-\sigma_0^{(1)})k}{(\sigma_0^{(1)}+\sigma_0^{(2)})^2B^2}
$$

It is not difficult to verify the fact that in the case of instability ($E_0B(\sigma_0^{(2)}-\sigma_0^{(1)})>0$) the most dangerous oscillations propagate along the magnetic field. Below we analyze precisely this case (i.e., $k_y = 0$) throughout.

The dependence $N_1(K_1)$ is shown for $\sigma_0^{(1)} = 0$ in Fig. 1. The crosses denote the results of the exact solution of the problem on the basis of Eq. (2.2). Thus, the chosen approximation for the velocity perturbation yields fully satisfactory results when the magnetic field is taken into account. In the statement considered (μ_0 = const) the approximation indicated evidently also yields good results when the viscosity is considered [11].

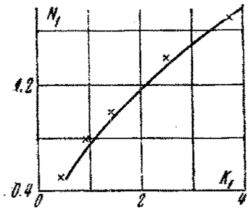

Fig. 1

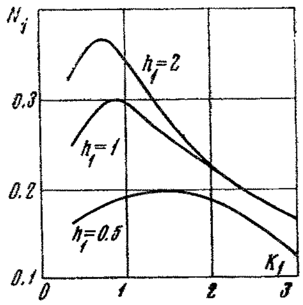

Fig. 2

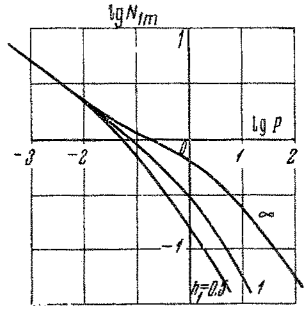

Fig. 3

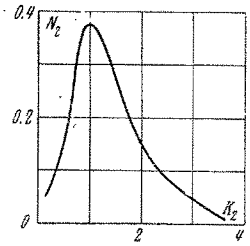

Fig. 4

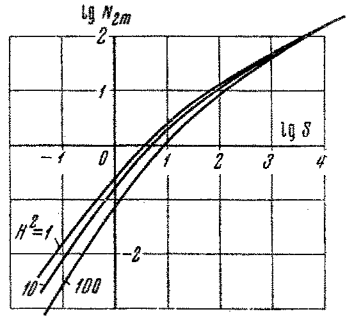

Fig. 5

Let us return to Eq. (4.1) for $h^{(1)} = h^{(2)} = h$ and $k_y = 0$ and let us write it in dimensionless form:

$$N_1^2 + \left(\frac{1}{2} - \frac{K_1 h_1}{\operatorname{sh} 2K_1 h_1} + 2K_1^2 P\right) N_1 - K_1 \operatorname{th} K_1 h_1 = 0$$

where

$$P = \frac{\mu_0 B^4 (\sigma_0^{(1)} + \sigma_0^{(2)})^3}{2\rho_0^2 E_0^2 (\sigma_0^{(2)} - \sigma_0^{(1)})^2}, \quad h_1 = \frac{hB^3 (\sigma_0^{(1)} + \sigma_0^{(2)})^2}{2\rho_0 E_0 (\sigma_0^{(2)} - \sigma_0^{(1)})}$$

It is not difficult to obtain the asymptotic expressions for the maximum growth rate N_{1m} of the stability as a function of P.

For the case $h_1 \to \infty$ we shall have

$$N_{1m} \to {}^1/_2 P^{1/3} \quad \text{for} \quad P \to 0$$
$$N_{1m} \to {}^2/_5 P^{1/5} \text{ for } P \to \infty$$

If h_1 is finite, then for $P \to 0$ we obtain the previous expression for N_{1m}; however, if $P \to \infty$, then $N_{1m} \to h_1/2P$.

The dependence $N_1(K_1)$ for this case is displayed in Fig. 2 for $P = 1$. Figure 3 shows the dependence of the maximum instability growth rate on the parameter P for various values of h_1. The stabilizing effect of the walls on the stability of the interface between media is evident.

5. Assume now that the density, conductivity, and viscosity of a medium which is confined between free surfaces spaced a distance d apart are distributed linearly:

$$\rho_0 = \rho_{00}(1 + \beta_1 z), \quad \sigma_0 = \sigma_{00}(1 + \beta_2 z)$$
$$\mu_0 = \mu_{00}(1 + \beta_3 z), \quad (|\beta_i d| \ll 1)$$

Then the solution of the original equation (1.2) may be written as follows:

$$W = A \sin \frac{m\pi}{d} z, \text{ where } m = 1, 2, \ldots$$

and the dispersion relationship will have the form

$$N_2^2 + 2\left[(m^2 + K_2^2) + \frac{k_x^2}{k^2} \frac{H^2}{(m^2 + K_2^2)}\right] N_2 - \left[G + \frac{k_x^2}{k^2} F\right] \frac{K_2^2}{m^2 + K_2^2} = 0$$

where

$$N_2 = \frac{2 d^2 n \rho_{00}}{\pi \mu_{00}}, \quad K_2 = \frac{dk}{\pi}, \quad H^2 = \frac{B^2 d^2 \sigma_{00}}{\pi^2 \mu_{00}}, \quad G = \frac{4 g \beta_1 d^4 \rho_{00}^2}{\pi^4 \mu_{00}^2}, \quad F = \frac{4 E_0 B \beta_2 d^4 \rho_{00}^2}{\pi^4 \mu_{00}^2}$$

The graph of $N_2(K_2)$ for $H^2 = 1$, $k_y = 0$, $G + F = S = 1$ is shown in Fig. 4.

Let us again consider the case of system instability (i.e., $S > 0$): under these conditions the most dangerous oscillations are propagated, as previously, along the magnetic field. It is not difficult to show that N_2 reaches a maximum at $m = 1$, and therefore below it is precisely this case which is analyzed. Figure 5 shows the dependence of the maximum instability growth rate on the parameter S for various values of the Hartmann number H. It is evident that as the Hartmann number increases the system becomes stabilized, although this stabilizing effect is comparatively small and is manifested at values of S which are not too large.

LITERATURE CITED

1. J. D. Jukes, "On the Rayleigh–Taylor problem in magnetohydrodynamics with finite resistivity," J. Fluid Mech., 16, Part 2 (1963)
2. R. Hyde, "Waves in a heavy viscous, incompressible, electrically conducting fluid of variable density in the presence of a magnetic field," Proc. Roy. Soc., 233A, No. 1194 (1955).
3. K. I. Kim, Rayleigh-Taylor Instability in Liquid-Metal Synchronous Magnetohydrodynamic Generators and Methods of Stabilizing It, Electr. MHD, Vol. 3, Vienna (1968).
4. H. P. Furth, J. Killeen, and M. N. Rosenbluth, "Finite-resistivity instabilities of a sheet pinch," Phys. Fluids, 6, No. 4 (1963).
5. S. I. Braginskii, "On the magnetohydrodynamics of weakly conducting fluids," Zh. Éksperim. i Teor. Fiz., 37, No. 5 (1959).
6. A. Lemaire, Instabilité de Rayleigh-Taylor sous L'éffet de Forces Electromagnetiques, Conf. Internat. Phenomenes Ionisat. Gaz., No. 6, Paris (1963).
7. Nguyen Duc, "Study of instability of the Rayleigh–Taylor type in magnetohydrodynamics, in: Direct Conversion of Thermal Energy into Electrical Energy, No. 7, [in Russian] VINITI (1968).
8. R. Devine, C. Karr, H. Lecroart, C. Marle, X. Nguyen Duc, J. Malpat, and R. Forte, Evaluation des Éffets de L'instabilité de Rayleigh–Taylor et de la Diffusion Turbulente dans la Conversion en Veine Inhomogene, Electr. MHD, Vol. 3, Vienna (1966), p. 501.
9. Lord Rayleigh, "Investigation of the character of equilibrium of an incompressible heavy fluid of variable density," Proc. London Math. Soc., 14, 170-177 (1883).
10. S. Chandrasekhar, "The character of the equilibrium of an incompressible heavy viscous fluid of variable density," Proc. Cambr. Phil. Soc., 51, Part 1 (1955).
11. W. H. Reid, "The effects of surface tension and viscosity on the stability of two superposed fluids," Proc. Cambr. Philos. Soc., 57, Part 2 (1961).